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Abstract

A pressure-based multi-block computational method is developed for solving the incompressible Navier-Stokes equations in general curvilinear grid systems. The scheme is based on the semi-implicit type flow solver with the staggered grid. Issues concerning the mass and momentum flux treatments at the discontinuous grid interface are addressed. Systematic numerical experiments for different interface treatments involving (i) straightforward interpolation, (ii) globally conservative scheme, and (iii) locally conservative scheme have been conducted. It is demonstrated that mass conservation has to be maintained locally, at the grid interface, with accuracy compatible with that of the scheme used in interior domain. Direct interpolation or globally conservative interface treatment of mass flux can not yield solutions with desirable accuracy.

1. Introduction

The numerical solution of fluid flow equations generally requires the generation of a grid for the region of interest. For many engineering problems with complex geometries, the generation of a single structured grid to cover the whole domain with the desirable grid distribution can be very difficult. For some flow problems with multiple length scales, it is also hard to generate a single structure grid to resolve all the flow features with reasonable grid points. These difficulties can be overcome to a limited extent by applying sophisticated grid generation schemes to construct a single grid with suitable characteristics. However, the degree of satisfaction achievable with such a process is highly problem dependent. To simplify this problem, it is becoming more common to use several grids at once, each with a regular grid structure. The various grids may either overlap in an irregular fashion or patch together. Each component grid covers a relatively simpler geometry and can be generated individually. Since grid lines need not be continuous across grid interfaces, local grid refinement and adaptive redistribution can be conducted more easily to improve the solution accuracy. In the region with high flow gradients, by using such flexible grid layouts, the total storage and CPU time can be reduced while achieving the desirable solution accuracy.

In order to make good use of the multi-block method, one needs to appropriately handle the grid interface treatment associated with the flow solver. Because the grid lines may not be continuous across the block interfaces, interpolation and data communication methods have to be devised to transfer the information between blocks. These methods should be preferably easy to implement while maintaining good efficiency and desirable accuracy. Besides these requirements, for many fluid flow problems containing varying flow gradients, it is often important to use the conservative interface procedure to ensure that the physical laws are satisfied there [1,2]. Clearly, these considerations impose serious constraints on the construction of an interface scheme; furthermore, in some cases the above requirements can conflict with one another. Simultaneous achievement of both conservation and accuracy can be a very difficult task.

Some progress has been made in this area, with different goals, for different fluid physics and multi-block arrangements, including patched and overlapped grids. Patched grids are individual grid blocks of which two neighboring blocks are joined together at a common grid line without overlap. With overlapped grids the grid blocks are partially superimposed on each other to cover the region of interest. Patched grid is relatively easy for data structure management, but it still has some difficulty with grid generation because of the interface constraints. Overlapped grid has more flexibility with grid generation, but, in general, its data management is more complex. For both grids arrangements, issues of interface treatment regarding both conservation laws and spatial accuracy need to be addressed. In the compressible flow regime, Rai [3,4] has developed conservative interface schemes for Euler equations calculation on the patched grid in a general curvilinear coordinate framework, for both explicit and implicit time integration schemes. Chesshire and Henshaw [5] have developed an overlapped grid generation method and a set of data structure, and solved the compressible Navier-Stokes equations. They treat the grid interfaces using interpolation without fluxes conservation. Meakin [6] has investigated the spatial and temporal accuracy of overlapped grid methods for inviscid moving body problems, using tri-linear interpolation for grid interface treatment. He suggests

that grid resolution is the primary issue. Attempts have been made by Moon and Liou [7] and Wang and Yang [8] to devise conservative interface schemes for overlapped grids. However, the issues of the importance of conservative interface treatment versus solution accuracy, and the requirements in local and global conservation have yet been clearly addressed.

In the incompressible flow regime, some progress has also been achieved. Hinatsu and Ferziger [9] have solved the unsteady Navier-Stokes equations in complex geometry using an explicit method. They do not enforce the mass conservation at the grid interface. Yung et al. [10] have solved the Navier-Stokes equations with a semi-implicit algorithm in the patched curvilinear grid system. However, the fluxes in general are not treated conservatively across grid interface. Lai et al. [11] have solved Navier-Stokes equations with the patched curvilinear grid system. They treat all the diffusion terms implicitly across the grid interfaces by defining individual nodal points for each block in the common areas. This way, while easy to implement, does not guarantee the satisfaction of the conservation laws across interface either. Meakin and Street [12] have solved the unsteady Navier-Stokes equations using the overlapped grid and applied the method to three-dimensional environmental flow problem. Tu and Fuchs [13] have developed a computational methodology for Navier-Stokes with emphasis on using an overlapped grid technique and multigrid method, and applied the method to the unsteady three-dimensional internal engine flow simulation. Neither work enforces conservative interface treatment. Henshaw [14] has used a fourth-order accurate method to solve incompressible Navier-Stokes equations on overlapped grids. In his method, the discrete divergence of the velocity field is not exactly zero, and a damping term is needed to stabilize the computations. Wright and Shyy [15] developed a pressure-based multi-block method for solving the incompressible Navier-Stokes equations on domains composed of an arbitrary number of overlapped grid blocks in the Cartesian grid system. A locally conservative internal boundary scheme, with first order accuracy, is devised to ensure that global conservation of mass and momentum fluxes are maintained. This methodology has also been extended to the curvilinear coordinates [16]. Clearly, although much progress has been achieved in the composite grid with complex fluid flow problems computed, some important and fundamental issues still need to be investigated further. For example, for incompressible flow problems, the mass conservation should be strictly maintained over the entire flow boundary; otherwise, the compatibility condition will not be satisfied. For the multi-block method, with which the discontinuous grid interface can be introduced into the flow domain, the conservative treatment of the mass flux along the grid interface may be critical for obtaining a converged solution. However, a interface scheme of lower order accuracy than the interior nodes, although conservative, may not be desirable from the resolution viewpoint, either. These issues have not been thoroughly investigated. Furthermore, for incompressible flow problems, the pressure may be only known up to an arbitrary constant. When the solver is applied to the multi-block grid, the pressure in different grid blocks may be independent. The coupling of the pressure at the interface can affect the solution process.

In the present study, the issues of conservation and accuracy of the interface treatment in a multi-block method is investigated for incompressible viscous flow computations on general curvilinear staggered grids. Effects of non-conservative and conservative treatments, either only globally along the whole interface or locally for each computational cell, for mass flux across

the grid interface on solution accuracy are focused on. The methods are tested for a recirculating problem with multi-block grids. A variety of numerical experiments with different grid resolutions are conducted, and the results are compared with the benchmark solution to assess the issues involved. The momentum flux treatment involving convection, pressure and viscous terms at the interface for momentum equations are also discussed.

2. Governing equations and numerical algorithm

The governing equations adopted in this study are the 2-D steady state, incompressible, constant property, Navier-Stokes equations along with the corresponding form of the continuity equation.

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0 \quad (1a)$$

$$\frac{\partial}{\partial x}(\rho uu) + \frac{\partial}{\partial y}(\rho uv) = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial x}\left(\mu \frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) \quad (1b)$$

$$\frac{\partial}{\partial x}(\rho uv) + \frac{\partial}{\partial y}(\rho vv) = -\frac{\partial p}{\partial y} + \frac{\partial}{\partial x}\left(\mu \frac{\partial v}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu \frac{\partial v}{\partial y}\right) \quad (1c)$$

With the introduction of coordinate transformation $\xi = \xi(x, y)$, $\eta = \eta(x, y)$, the equations above are cast in the curvilinear coordinates,

$$\frac{\partial}{\partial \xi}(\rho U) + \frac{\partial}{\partial \eta}(\rho V) = 0 \quad (2a)$$

$$\frac{\partial}{\partial \xi}(\rho Uu) + \frac{\partial}{\partial \eta}(\rho Vu) = -y_\eta \frac{\partial p}{\partial \xi} + y_\xi \frac{\partial p}{\partial \eta} + \frac{\partial}{\partial \xi}\left[\frac{\mu}{J}(q_1 u_\xi - q_2 u_\eta)\right] + \frac{\partial}{\partial \eta}\left[\frac{\mu}{J}(-q_2 u_\xi + q_3 u_\eta)\right] \quad (2b)$$

$$\frac{\partial}{\partial \xi}(\rho Uv) + \frac{\partial}{\partial \eta}(\rho Vv) = +x_\eta \frac{\partial p}{\partial \xi} - x_\xi \frac{\partial p}{\partial \eta} + \frac{\partial}{\partial \xi}\left[\frac{\mu}{J}(q_1 v_\xi - q_2 v_\eta)\right] + \frac{\partial}{\partial \eta}\left[\frac{\mu}{J}(-q_2 v_\xi + q_3 v_\eta)\right] \quad (2c)$$

where

$$\begin{aligned} U &= uy_\eta - vx_\eta, & V &= vx_\xi - uy_\xi \\ q_1 &= x_\eta^2 + y_\eta^2, & q_2 &= x_\xi x_\eta + y_\xi y_\eta, & q_3 &= x_\xi^2 + y_\xi^2 \\ J &= x_\xi y_\eta - x_\eta y_\xi \end{aligned}$$

A staggered grid system is adopted, as shown in Fig. 1. The scalar variables are located at the center of the four adjacent grids. Both u and U are located at the midpoint of the east and west faces of the control volume. Both v and V are located at the midpoint of the north and

south faces of the control volume. In terms of the notation shown in Fig. 1, for a node p enclosed in its cell and surrounded by its neighbors n , s , e , w , the finite-difference approximation to the conservation laws can be performed by taking the integral of momentum equations over the control volume and discretizing it. By arbitrarily taking $\Delta\xi=\Delta\eta=1$, the resulting u - and v -momentum equations yield

$$[\rho U u + y_\eta p - \frac{\mu}{J}(q_1 u_\xi - q_2 u_\eta)]|_w^e + [\rho V u - y_\xi p - \frac{\mu}{J}(-q_2 u_\xi + q_3 u_\eta)]|_s^n = 0 \quad (3a)$$

$$[\rho U v - x_\eta p - \frac{\mu}{J}(q_1 v_\xi - q_2 v_\eta)]|_w^e + [\rho V v + x_\xi p - \frac{\mu}{J}(-q_2 v_\xi + q_3 v_\eta)]|_s^n = 0 \quad (3b)$$

The equations above can be put into a general difference form:

$$[\rho U \phi - \frac{\mu}{J}(q_1 \phi_\xi - q_2 \phi_\eta)]|_w^e + [\rho V \phi - \frac{\mu}{J}(-q_2 \phi_\xi + q_3 \phi_\eta)]|_s^n = S \cdot J \quad (4)$$

where ϕ is the general dependent variable and S is the source term. With appropriate finite difference schemes representing the convective and diffusive terms at the control volume boundaries, the discretized equation relating the variable at a central point p and its neighboring values is obtained [17],

$$a_p \phi_p = a_e \phi_e + a_w \phi_w + a_n \phi_n + a_s \phi_s + S_p \quad (5)$$

where a 's are the coefficients resulting from the numerical schemes chosen in course of discretization. The pressure term and cross-derivative portion of the viscous terms due to the non-orthogonal coordinates are treated here as the source term S_p . The continuity and momentum equations can be used to formulate a pressure correction equation. The pressure correction, p' , is used to update the pressure field, and in conjunction with a velocity correction formula to obtain a velocity field satisfying the continuity equation at convergence. The discretized form of the pressure correction equation is presented as:

$$a_p p'_p = a_e p'_e + a_w p'_w + a_n p'_n + a_s p'_s + S_p \quad (6)$$

$$a_p = a_e + a_w + a_n + a_s$$

$$S_p = (\rho U^*)_w - (\rho U^*)_e + (\rho V^*)_s - (\rho V^*)_n$$

where the superscript $*$ designates the intermediate solution, and p' designates the correction made. A detailed discussion of this algorithm can be found in Refs. [17,18] and will not be

repeated here. In single-block grid computations, the solution procedure is as follows: (a) The momentum equations are solved to obtain the Cartesian velocity components with the given pressure field. When solving the momentum equations, the contravariant velocity components U and V are calculated after updating each of the velocity components. (b) With the updated U and V , the pressure correction equation is solved to obtain p' . (c) The velocity and pressure fields are updated with the solution of the pressure correction equation. Procedures (a)-(c) are repeated until the momentum and continuity equations are simultaneously satisfied to the required degree of accuracy. The procedure above is a semi-implicit procedure, which is reduced to the SIMPLE procedure if the Cartesian grid is adopted [19]. With curvilinear coordinates, a combined use of both Cartesian and contravariant velocity components is devised. In particular, while the Cartesian components are the primary dependent variables in momentum equations, the contravariant components are corrected first in the pressure correction step to ensure the satisfaction of mass continuity. Because of the semi-implicit nature of the procedure, when the algorithm is applied to the multi-block grid computation, different computational strategies between different grid blocks can be adopted: (1) Within each grid block, procedures (a)-(c) are repeated several times to update the solutions without updating the boundary and interface. Afterwards, the computations are conducted in the neighboring blocks. Such a block to block cycling procedure continues until convergence is achieved in all blocks. (2) While solving each of the u -, v -momentum and pressure correction equations, iteration is conducted from block to block, without updating other equations. In other words, procedures (a)-(c) are conducted in the outer loop with the multi-block computation embedded within each differential equation. In this study, strategy (1) is adopted, since it allows the interface treatments among different equations to be handled together. The computation continues until the mass and momentum residuals, normalized by the mass and momentum fluxes entering the whole domain, in each block meet the convergence criterion. More information regarding both the composite grid techniques and the pressure-based algorithm discussed here can be found in Shyy [20].

3. Grid interface treatment

The basic arrangement in the present multi-block grid system is that there is at least one grid layer overlap, for each block, between the adjacent blocks. However, The grid lines from both blocks need not be continuous in the overlapped region. At grid interfaces, the boundary conditions for each block are needed in order to solve the equations. Because the grid interfaces are not the physical boundaries, the interface boundary values are not known *a priori* and must be obtained as a part of the whole solution. Some interpolations are needed to acquire the intermediate interface boundary conditions between neighboring blocks. Then, a block to block iteration is conducted to obtain the solution in the whole domain.

3.1 Interface treatment for the momentum equations

For the momentum equations, e.g., the u -momentum equation, the fluxes

$$E|_{e,w} = \rho Uu + y_{\eta} p - \frac{\mu}{J} (q_1 u_{\xi} - q_2 u_{\eta}) \quad (7a)$$

$$F|_{n,s} = \rho V u - y_{\xi} p - \frac{\mu}{J} (-q_2 u_{\xi} + q_3 u_{\eta}) \quad (7b)$$

could be interpolated directly from neighboring blocks at the interfaces. Since they include velocity gradient and metric terms, such a direct interpolation does not result in a conservative solution. In the present treatment, direct interpolation of dependent variables, with adjustment of pressure to maintain the global momentum conservation at grid interfaces, is adopted. The quadratic or the linear interpolation can be used [5]. With the staggered grid arrangement, the interface treatments for the east and west boundaries are different from those for the north and south boundaries. Here, the east boundary is defined as the grid boundary with grid index $i=i_{\max}$, and the west boundary as with index $i=1$. The north and south boundaries are the other sides with $j=j_{\max}$ and $j=1$ respectively.

First consider the evaluation of the flux E at the east or the west interface. Due to the utilization of the staggered grid, the control volume face at which the u -momentum flux is evaluated is not at the grid boundary, as illustrated in Fig. 2. The geometric quantities q_1 , q_2 and J are well defined within the current block. The velocity component u at the grid interface boundary is interpolated from the neighboring block to evaluate the viscous term in E . The pressure term at the control volume face is also available within the current block.

Second, consider the evaluation of the flux F at the north or south interface. The control volume face coincides with the grid interface. The geometric quantities and the viscous terms are not well defined within the block. With present treatment, the interface grid lines are extended from the original block to intersect with the neighboring grid lines to form fictitious interpolation points as illustrated in Fig. 3. Then the geometric quantities are obtained based on linear interpolation. The u components are interpolated at the fictitious points and both convection and viscous terms can be evaluated from the values obtained from the original block and the fictitious nodes. Generally, for curvilinear grid, the pressure term will appear in the flux F if y_{ξ} is not zero. Because the pressure is not defined at the control volume face, either it can be extrapolated from inside the current block or interpolated from the adjacent block. The latter one is adopted in this study. With the present solver, the boundary condition for the pressure correction equation is of Neumann type, which means the pressure is known up to an arbitrary constant. In the present approach, a pressure adjusting method is used to connect the pressure fields of different blocks consistently, which is based on the total momentum flux balance across the interface [15]. This adjustment is conducted between the block to block iterations to keep the pressure fields compatible between the blocks. The interface treatment above also can be applied to the v -momentum equation similarly. It is noted that with the present pressure adjustment procedure, the total momentum fluxes across the grid interface are uniquely determined for both blocks, resulting in a conservative treatment.

3.2 Interface treatment for the mass flux

In general, two types of boundary conditions can be used for the pressure correction equation. If the pressure is known at the boundary, the value of the pressure correction there is, of course, zero. If, instead of the pressure itself, the velocity component normal to the boundary

is prescribed (such as the inflow and solid wall conditions), then, due to the staggered grid arrangement, the Neumann type of boundary condition is imposed on the pressure correction. The value of pressure correction at the boundary is no longer needed. Consider the pressure correction Eq. (6), suppose the normal velocity component is known at the south boundary, the equation for a control volume next to that boundary becomes

$$a_p p'_p = a_e p'_e + a_w p'_w + a_n p'_n + S_p \quad (8a)$$

$$a_p = a_e + a_w + a_n \quad (8b)$$

$$S_p = (\rho U^*)_w - (\rho U^*)_e + (\rho V)_s - (\rho V^*)_n \quad (8c)$$

Where $(\rho V)_s$ is the mass flux from the south boundary and is known, U^* and V^* are the intermediate values to be further corrected by the pressure correction p' . As stated above, p'_s does not appear because no "correction" is necessary on the south boundary. Since the continuity equation is not solved explicitly, the outflow boundary condition should always satisfy the global mass conservation. Usually, a global mass correction is applied to outflow boundary during the solving process to help maintaining the global mass conservation. This correction will not affect the final solution when the process is finally converged. Otherwise, the process will either converge very slowly or even fail to converge [21]. Now consider the case that the south boundary is the grid interface. In this situation, $(\rho V)_s$ is not known *a priori* during the solution process. It can be obtained from the intermediate solution in the overlapped region of the neighboring block at the previous iteration. But with the discontinuous grid interface, direct interpolations generally will not satisfy the mass conservation across the grid interface. Non-conservative error will be introduced at the grid interface and the magnitude of error will depend on the order of the interpolation method used at the grid interface. In this study, the linear and quadratic non-conservative interpolations are implemented to investigate the effect of non-conservative error from the grid interface on the solution. Also, several conservative treatments of different local and global natures, and of different orders of accuracy are implemented and are tested.

(a) Interpolation with global correction:

First, a non-conservative interpolation is used to obtain the mass flux at the interface from the neighboring block. The total flux evaluated from neighboring block is denoted as F_n , the total interpolated flux is denoted as F_e . Second, the mass deficit $\Delta F = F_n - F_e$ is computed. Then the mass flux at each current interface control volume face is added with $\Delta F/N$ (suppose there are N control volumes on the interface) so that the total flux across the interface is conserved globally. But this conservation is not enforced locally, and its effect on the solution in the whole domain needs to be assessed.

(b) Piece-wise constant interpolation: Such a scheme is used in Refs. [3,15], which by nature is locally conservative, but with only first-order accuracy.

(c) Linear (or quadratic) interpolation of mass flux with local conservative correction:

Suppose a portion of an interface, corresponds to the width of a single control volume of the coarse grid 1, and to the width of several control volumes of the fine grid 2, indexed from $i=1$ to $imax$. This scenario is shown schematically in Fig. 4, where \bar{V}_c is the contravariant velocity component normal to the grid face, normalized by the control volume face length S_c , at the coarse grid control volume face, and \bar{V}_{fi} represents the contravariant velocity component, normalized by the control volume face length S_{fi} , at the fine grid control volume face. From fine grid to coarse grid, \bar{V}_c can be obtained as

$$\bar{V}_c S_c = \sum_{i=1}^{imax} \bar{V}_{fi} S_{fi} \quad (9)$$

Equation (9) is for the constant density flow. It can be easily extended to account for density variation. For the sake of simplicity, this aspect is neglected in the current discussion. In this way, the flux into the coarse grid is uniquely determined from the corresponding fine grid fluxes conservatively. Conversely, given the coarse grid flux $\bar{V}_c S_c$, the conservation constraint yields the follows:

$$\sum_{i=1}^{imax} \bar{V}_{fi} S_{fi} = \bar{V}_c S_c \quad (10)$$

In this situation, conservation does not provide unique values for the \bar{V}_{fi} . A certain distribution has to be chosen to decide each \bar{V}_{fi} . As a first approximation, \bar{V}_{fi} is obtained using the linear (or quadratic) interpolation of the normalized contravariant velocity in the coarse grid. The interpolated value is denoted as \bar{V}'_{fi} . With the first approximation, Eq.(9) is not satisfied. Then the fine grid fluxes are scaled so that the total flux obtained is $\bar{V}_c S_c$. Accordingly, the values \bar{V}_{fi} at the fine grid boundary are computed as follows:

$$\bar{V}_{fi} S_{fi} = \frac{|\bar{V}'_{fi}| S_{fi}}{\sum_{i=1}^{imax} |\bar{V}'_{fi}| S_{fi}} \bar{V}_c S_c \quad (11)$$

From Eq.(11), it can be seen that Eq.(9) expressing flux conservation from coarse grid to fine grid is satisfied and the flux distribution is close to that determined by linear (or quadratic) interpolation.

If the grid interface is not exactly matched in the coarse to fine grid manner, as illustrated in Fig. 5, then, a split-merge procedure is devised in the above mentioned spirit, as shown in Fig. 6. The control volume in grid 2 can be split into smaller subcontrol volumes. In this manner, from control volumes in grid 1 to subcontrol volumes in grid 2, the locally conservative treatment can still be applied on a cell-by-cell basis, after the interpolation has been conducted. After that, the fluxes at the split control volume faces will be merged back to get the flux at the original control volume face. From grid 2 to grid 1, this same split-merge treatment is also applicable. The only extra work in this procedure is to create some arrays to store the intermediate information. Thus, the interface can be treated no matter what kind of interface arrangement is encountered. This linear (quadratic) interpolation with local correction treatment is not limited to the mass flux, it also can be applied to the momentum flux conservative treatment.

3.3 Data structure

Because of the possible grid discontinuity, a set of data structure has to be devised to deal with information transfer between different grid blocks. The data structure should not only provide the necessary information for the chosen interface treatment but also be as simple as possible. In the present method, all the data structure information is based on the grid distribution. The minimum requirement for the grid interface between the two adjacent grid blocks is that there exists at least one grid layer overlap. For each block, each side of the grid boundaries may be divided into several segments corresponding to different boundary condition types. For each segment, the indices of the starting and the ending grid points, the boundary condition type and the corresponding values of the dependent variables on the physical boundaries are assigned. If the segment is a grid interface, the block number of the adjacent block, the identification of overlap direction (in ξ or η direction), and the number of overlap layers are assigned as well. Based on the input above, a series of intersection tests are conducted to provide the information for the coefficients for interpolation and conservative treatment. These coefficients are stored for later use.

4. Numerical experiments and discussions

The test problem is the lid-driven cavity flow problem with $Re=1000$. First, a 3-block discontinuous Cartesian grid configuration with different grid resolutions is used. A grid system of 41×21 , 81×13 and 41×11 grid points for block 1, 2 and 3, respectively, as shown in Fig. 7 is first employed. Here, the interface between block 1 and 2 coincides with the cavity horizontal center line. The second grid system doubles the grid resolutions of the first system, and has 81×41 , 161×23 and 161×21 grid points in each block, respectively. The third grid system doubles the grid resolution in x direction of the second grid system and has 161×41 , 321×23 and 161×21 grid points in three blocks. To clarify the terminology, the composite grid for the whole flow domain is called a "grid system" here. Each grid system consists of several blocks. The first grid system described above is denoted the coarse grid system, the second the median grid system, and the third the fine grid system. The three grid systems share the same topological characteristics in each block. We have created these three systems to investigate the interplay of interface treatment and overall grid resolutions. The grid layouts are not ideally suitable for the present recirculating flow, they are purposely set up to test the relative merits of different

interface treatments. For all the test cases presented in this work, the second-order central difference is used for convection, diffusion and pressure terms.

Case 1. *The Cartesian velocity components u and v and the contravariant velocity components U and V are linearly interpolated at the grid interfaces. Total momentum fluxes across the interface are conserved via the pressure adjustment. No mass conservation is enforced across the grid interfaces.* The computations are conducted over the coarse, median and fine grid systems. For all three grid systems, the normalized residuals for u and v momenta are below 10^{-4} . The normalized mass residuals reach down to 1.5×10^{-3} , 3.3×10^{-4} and 1.3×10^{-4} for the coarse, median and fine grid systems, respectively, and stabilize at those levels. Figures 8a and 8b show the u -component distributions along the vertical center line and v -component distributions at the horizontal center line; they are compared with the corresponding benchmark solution reported by Ghia et al.[22]. It can be seen that the solutions for all the three grid systems have substantial discrepancies with respect to the benchmark values. Although in general, the solutions improve as the grids are refined, the overall performance of all three systems are unsatisfactory. It is noted that for this problem computed with single grid distributed uniformly, a 81×81 grid system can yield very accurate solution already [23]. Accordingly, it is unsatisfactory to observe that both median and fine grid systems, even with resolutions better than the 81×81 single uniform grid, still do not yield accurate solutions. Figure 8c shows the interface pressure distributions computed on block 1 and 2. The pressure in each block has been adjusted according to the total momentum flux balance at the interface. The absolute value of the pressure has no practical meaning. For a well converged solution, the pressures from different blocks are expected to have the same distributions at the interface; in the current case, even with the fine grid system, the discrepancy between the two pressure distributions is obvious.

Case 2. *Both the Cartesian velocity components u and v and the contravariant velocities U and V are quadratically interpolated, but still without enforcing mass conservation.* This procedure is of higher interpolation accuracy than case 1; however, both are without conservative treatment. The motivation here is to test the role played by the interpolation accuracy, and to investigate that for viscous flow, whether conservative treatments are necessary for obtaining satisfactory solutions. For all three grid systems, the residuals for the momentum equations are below 10^{-4} . The mass residuals stabilize at the level of 2.3×10^{-3} , 4.3×10^{-4} and 7.5×10^{-5} for the coarse, median and fine grid systems, respectively. Figures 9a and 9b show the u -component and v -component distributions respectively for the three grid systems. Overall, these solutions are more accurate than those shown in Fig. 8, and the solution on the fine grid system shows obvious improvement over the coarse and median grid systems. However, discrepancies still exist between the present and the benchmark solutions. Figure 9c shows the pressure distributions from block 1 and 2 at the interface. The discrepancy between the two distributions also still exist. From cases 1 and 2, it can be seen that the solution can be improved with the grid refinement. The overall solution accuracy is better with quadratic interface interpolation than with linear interpolation, which is expected because the quadratic interpolation increases the order of interpolation accuracy and should reduce the non-conservative error for mass flux across the grid interfaces. But even with the fine grid system, the solution of quadratic interpolation is still not satisfactory. Since with both interpolation schemes, the solutions are not

as accurate as the single continuous grid results, the source of this inaccuracy must come from the non-conservative interface treatments across discontinuous grid blocks.

Case 3. Since the non-conservative linear and quadratic interpolations for the contravariant velocities cannot lead to a satisfactory solution even for a very fine grid, a conservative interface treatment is then tested. In this case, *the Cartesian velocity components u and v are linearly interpolated. The contravariant velocities U and V are linearly interpolated first, followed by a global correction procedure for mass flux as discussed previously.* The computation is conducted for the coarse grid system only. The momentum and mass flux residuals reach to the level of 10^{-5} . The results are shown in Figs. 10a, 10b and 10c. Obviously, the solution is not satisfactory. It appears that the conservative interface treatment conducted at the global level does not improve the solution accuracy.

Case 4. *The Cartesian velocity components u and v are still linearly interpolated. The contravariant velocities U and V are interpolated based on the piecewise constant formula, which by nature is locally conservative with first-order accuracy.* This treatment is implemented to investigate the effect of the local conservation on the solution. The computations are conducted for the three grid systems. For the coarse and median grid systems, the residuals for the momentum and mass fluxes reach down to 10^{-5} . But for the fine grid, the solution does not converge. Figures 11a and 11b present the u and v component distributions for the coarse and median grids. Both u and v profiles agree well with the benchmark solutions. Figures 11c and 11d exhibit the pressure distributions at the interfaces of block 1 and 2 for two grid systems. The pressures in the interface region obtained on different blocks follow each other generally well, except that high wave number oscillations appear on block 2. The cause of this nonphysical oscillation comes from the fact that the mass flux distributions in the overlapping region of the fine grid cells are assigned according to the piecewise constant formula. This distribution formula results in a series of stair-step mass flux profile on the fine grid block, forcing the pressure field to oscillate in response to the non-smooth mass flux distribution. This same reason is also probably responsible for the nonconvergence of the fine grid system.

Case 5. *The Cartesian velocity components u and v are linearly interpolated. The contravariant velocities U and V are linearly interpolated first, followed by a local correction to maintain the cell-by-cell mass conservation across the interfaces.* The residuals for momentum and mass fluxes reach down to 10^{-5} . Figures 12a and 12b show the u and v component distributions for three grid systems. The solution for the coarse grid system shows a very small discrepancy compared to the benchmark solution; the solutions on the median and fine grid systems agree very well with the benchmark solution. Figure 12c displays the pressure distributions at interface of block 1 and 2 for the fine grid system. The pressure distributions from the two adjacent grid blocks conform to each other very well. Clearly, local conservation holds a key to produce satisfactory solution accuracy.

Case 6. *The Cartesian velocity components u and v are quadratically interpolated. The contravariant velocities U and V are quadratically interpolated first, and then a local correction is applied to maintain the mass conservation across the interfaces.* Again, the computations are conducted over the three grid system and the residuals are at the level of 10^{-5} . Figures 13a and

13b show the u and v -component distributions. Good agreements, comparable to Case 5 (Fig. 12) between the current solutions and the benchmark solution are observed. Figure 13c presents the pressure distributions at block 1 and 2 interface for the fine grid system. The pressure distributions at the interface also conform to each other very well. For the present flow problem both the linear and quadratic interpolation with local correction give the satisfactory solutions. Since the discretization scheme for the interior cells are second-order accurate, it appears that a linear interpolation (aided with the follow-up local conservation treatment) procedure is sufficient.

Case 7. Finally, the same flow problem is investigated with a 3-block curvilinear grid, which has 81×41 , 161×23 and 81×21 grid points for block 1, 2 and 3, respectively, as shown in Fig. 14. The interface treatment is the same as that in Case 5, and the momentum and mass residuals reach to 10^{-5} . Figure 15 demonstrates the u velocity component distribution at the vertical center line and compares it to the corresponding benchmark solution. Good agreement is obtained; illustrating the flexibility of the curvilinear grid system.

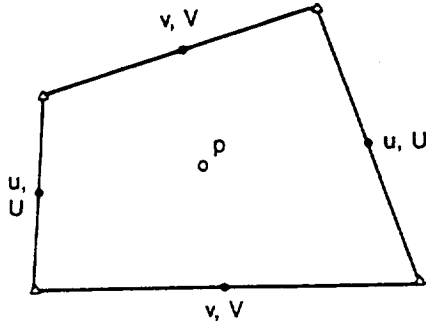
5. Conclusions

A pressure-based multi-block computational method has been developed for solving the incompressible Navier-Stokes equations in a general curvilinear grid system. For the momentum equations, the pressure fields between two adjacent blocks allow an arbitrary jump, which can be adjusted by conserving the total momentum fluxes across the block interface. The importance of maintaining local mass flux conservation across the interface with certain accuracy is illustrated through a series of numerical experiments. Specifically, both the linear and quadratic interpolations without conservative measure for the mass flux at the interface cannot lead to the desired solution even for very fine grid. Linear interpolation with global correction for mass flux does not improve the solution, either. The piecewise constant treatment can improve the solution accuracy due to its conservative nature, but creates artificial pressure oscillation due to non-smooth mass flux distribution in the fine grid block. Nevertheless, it illustrates the importance of local mass flux conservation across the interface. Both linear and quadratic interpolations, with local conservative correction prove to be good choices. The fact that their solutions are of very comparable accuracy indicates that (1) the interface treatment needs not to be of higher formal order of accuracy than the interior schemes, and (2) the local conservative mass flux treatment at interface holds a key for composite grid computations.

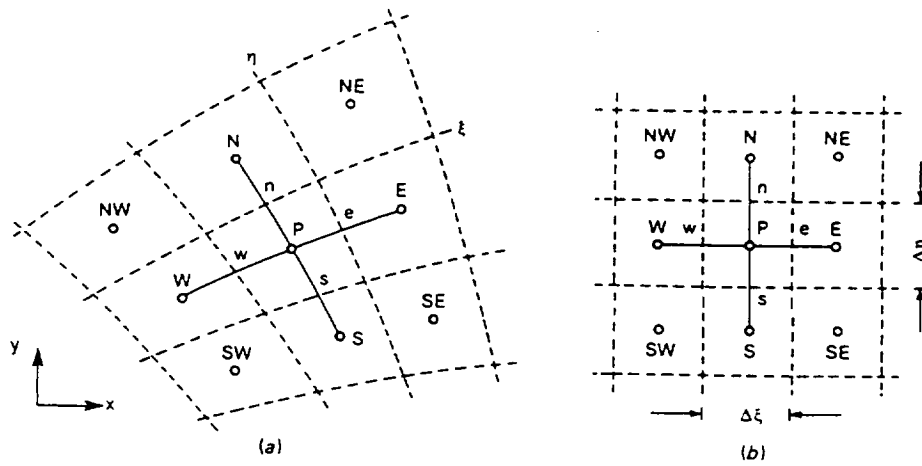
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(i) Configuration of a staggered grid system.



(ii) Finite-difference grid representation:
(a) physical plane; (b) transformed plane.

Fig. 1 Staggered grid and notation for both physical and computational domains

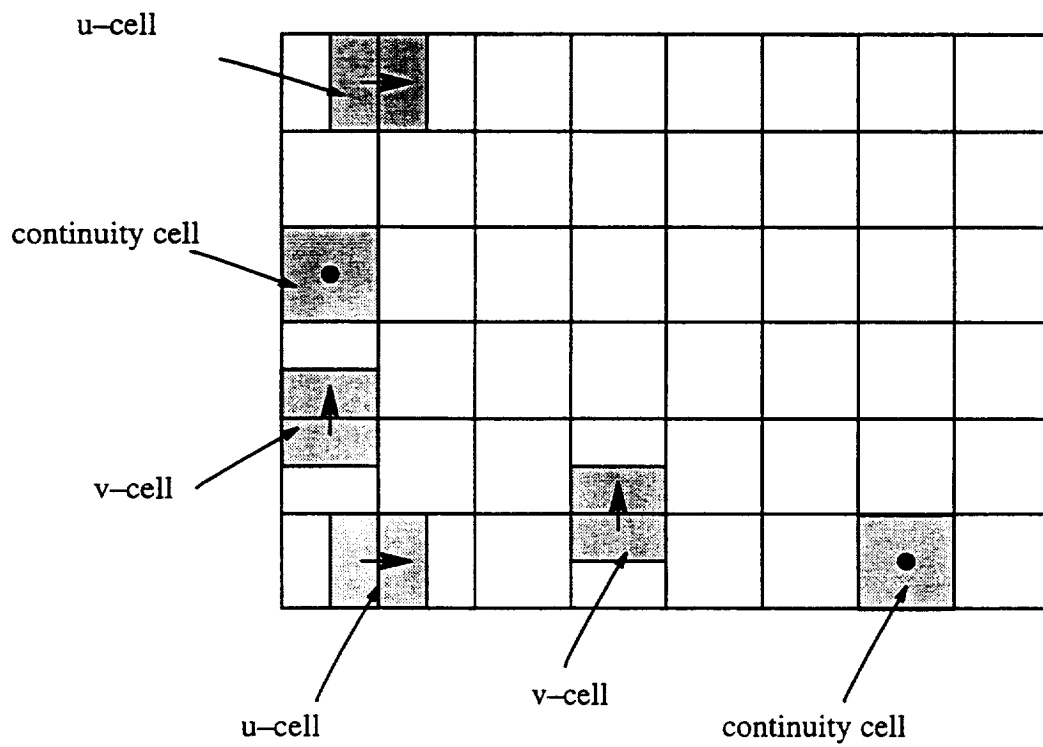


Fig. 2 Control volume and flux components in different governing equations and boundaries.

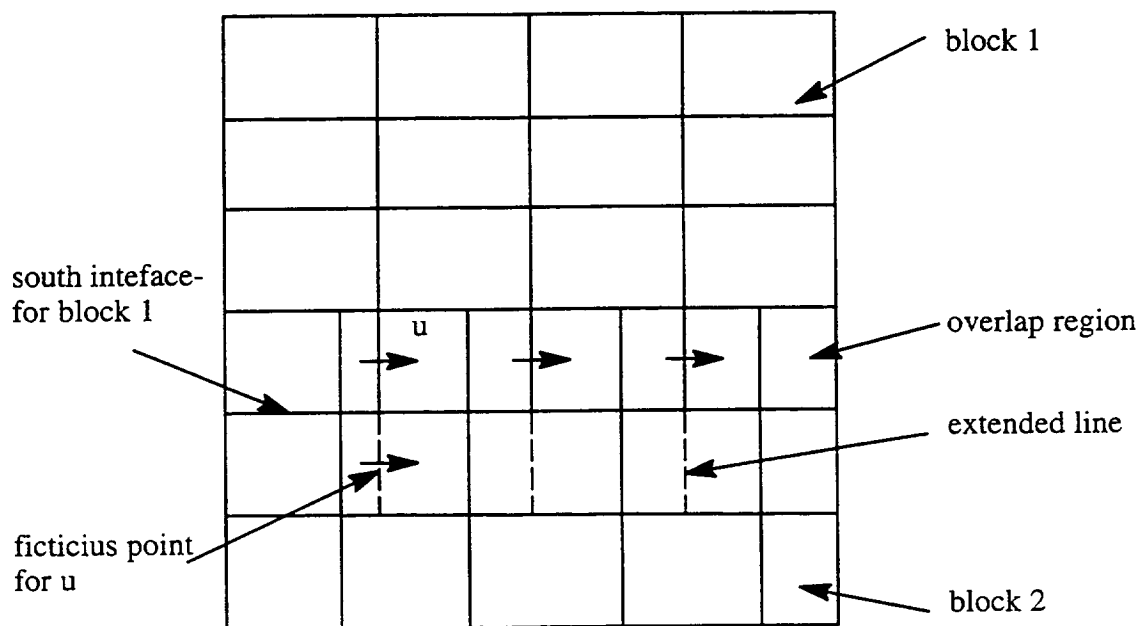


Fig. 3 The fictitious points arrangement for u at the south interface of block 1. The dashed lines are extended from the solid lines in block 1 and intersect with a grid line in block 2. These intersection points will be used to estimate the flux for block 1.

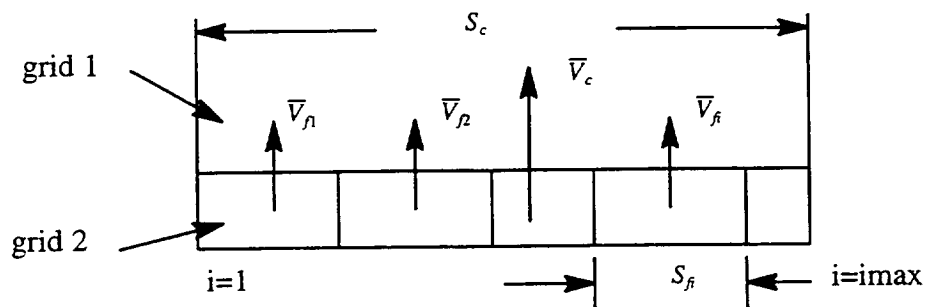


Fig. 4 Notations used in two-block interface (subscript c and f designate coarse and fine grid quantities, respectively)

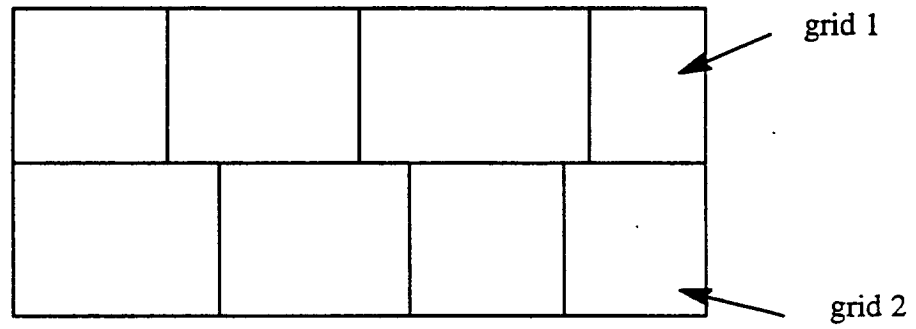


Fig. 5 General interface configuration: no distinct coarse or fine grid.

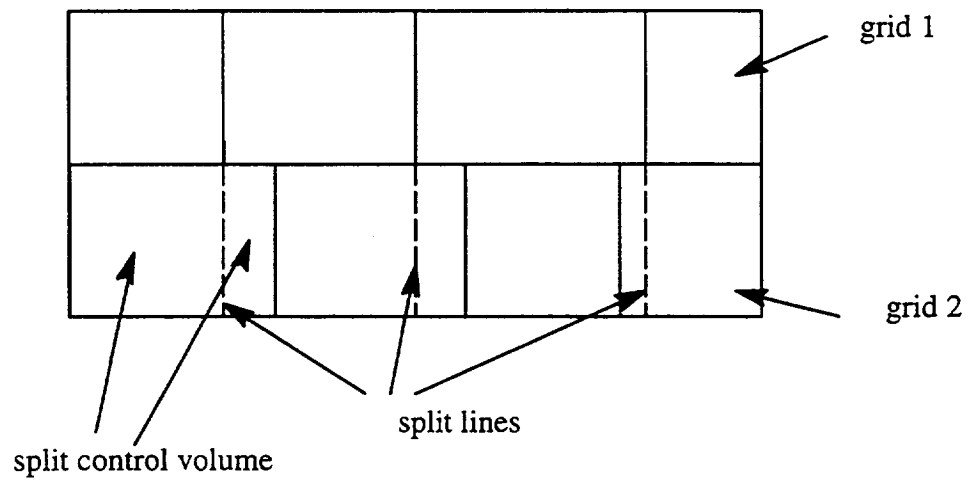


Fig. 6 Split control volume configuration: control volumes in grid 2 are split into subcontrol volumes

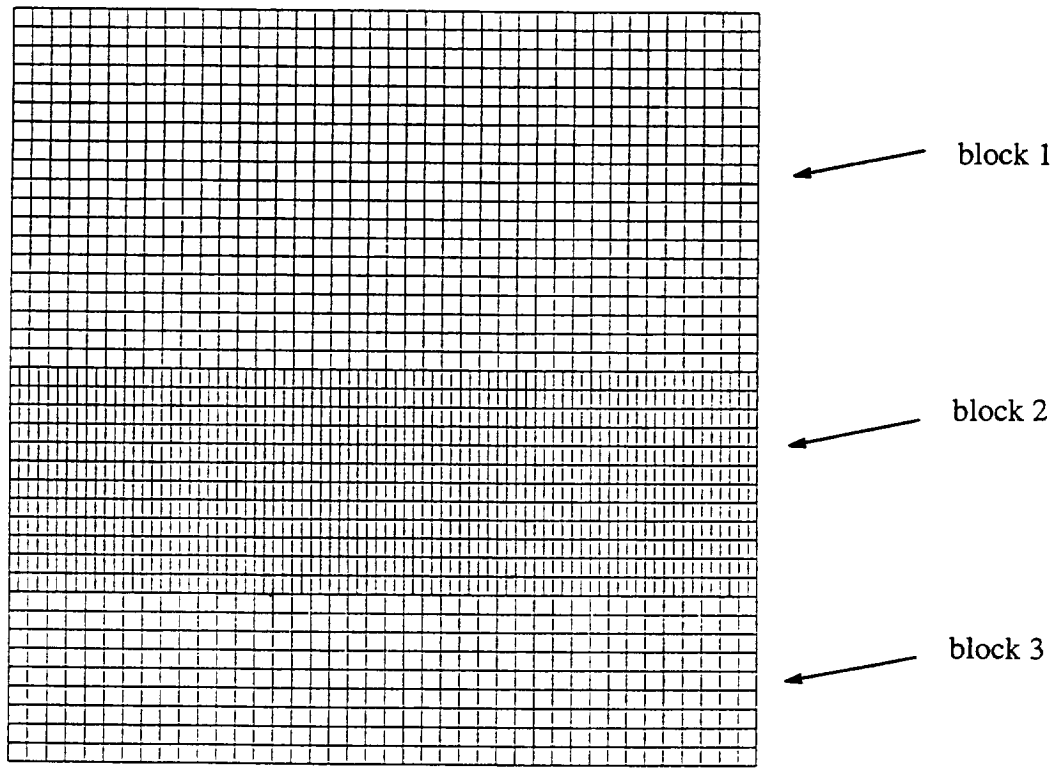


Fig. 7 Three-block grid with 41×21 , 81×13 and 41×11 nodes for block 1, 2 and 3, respectively.

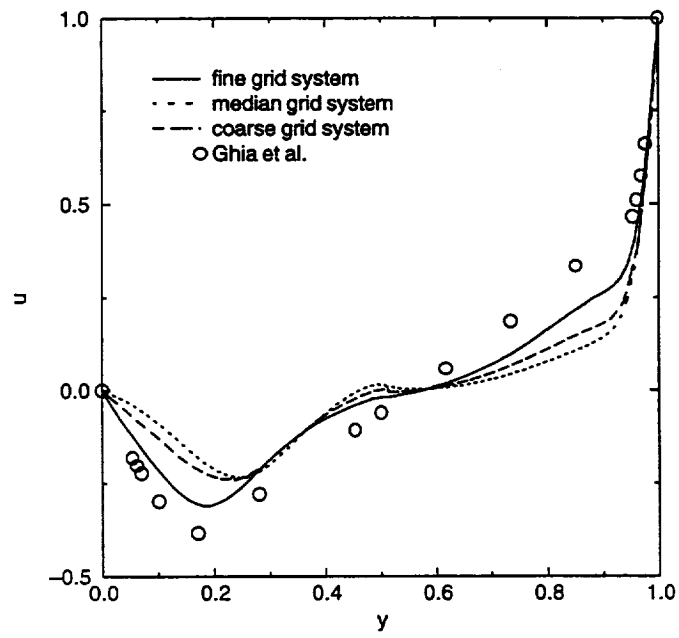


Fig. 8a The u velocity along the vertical center line

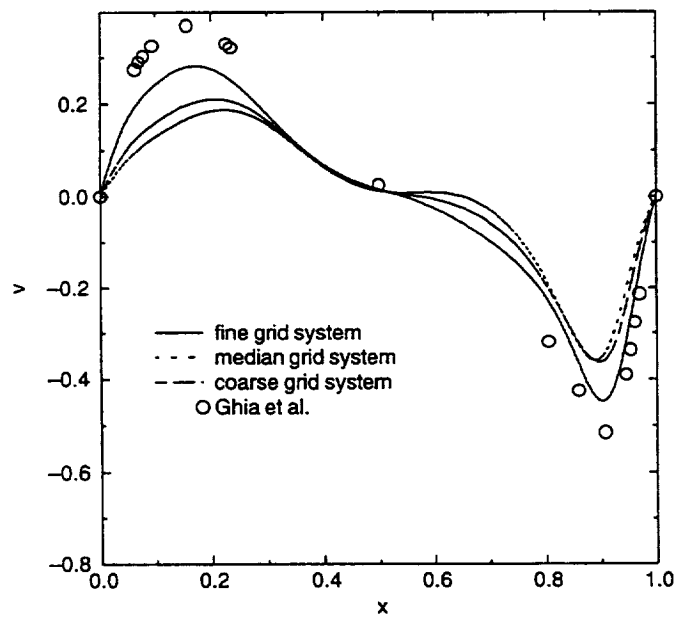


Fig. 8b The v velocity along the horizontal center line

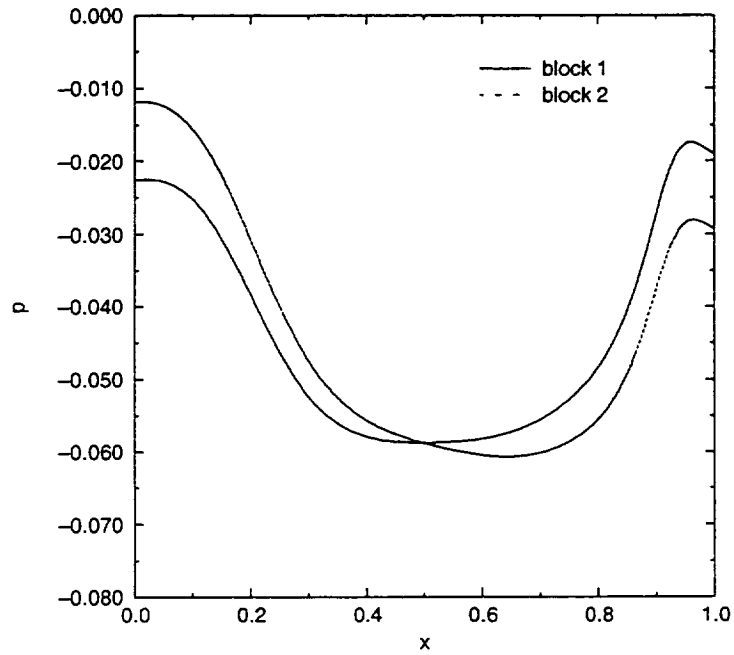


Fig. 8c The pressure distribution at the interface between block 1 and 2 on the fine grid system.

Fig. 8 Solution profiles based on linear interpolation for U and V without mass conservation correction.

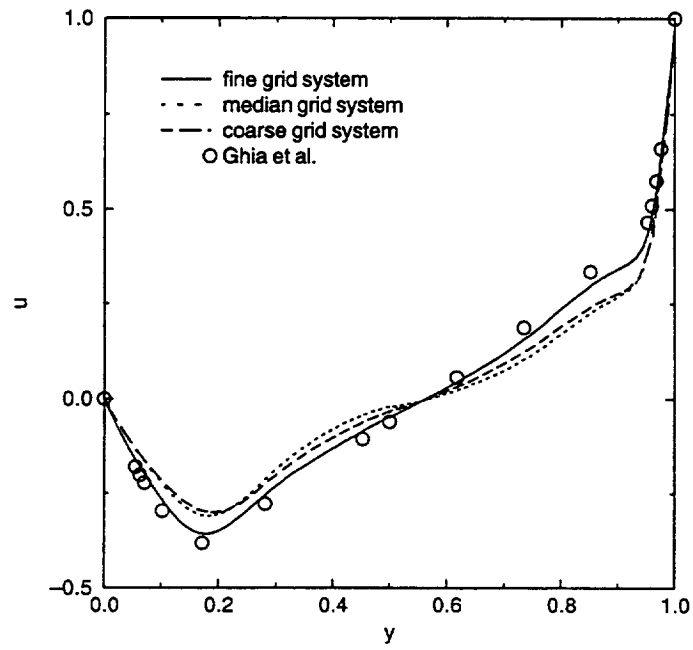


Fig. 9a The u velocity along the vertical center line.

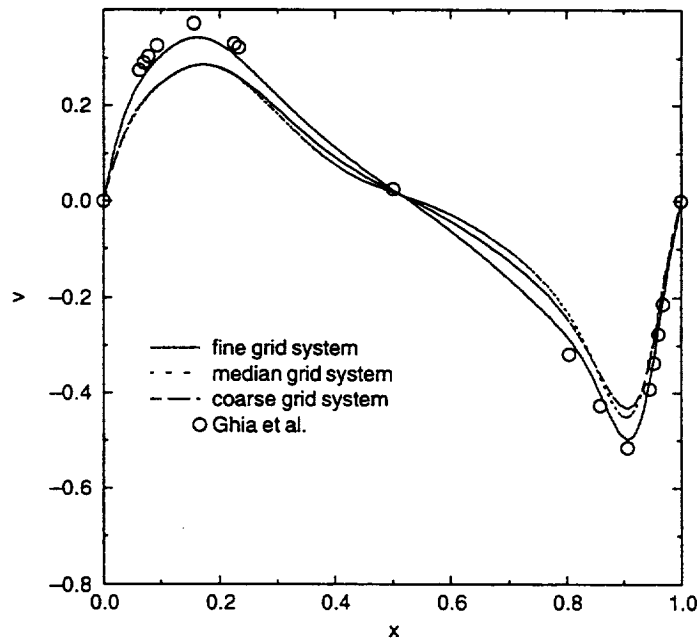


Fig. 9b The v velocity along the horizontal center line.

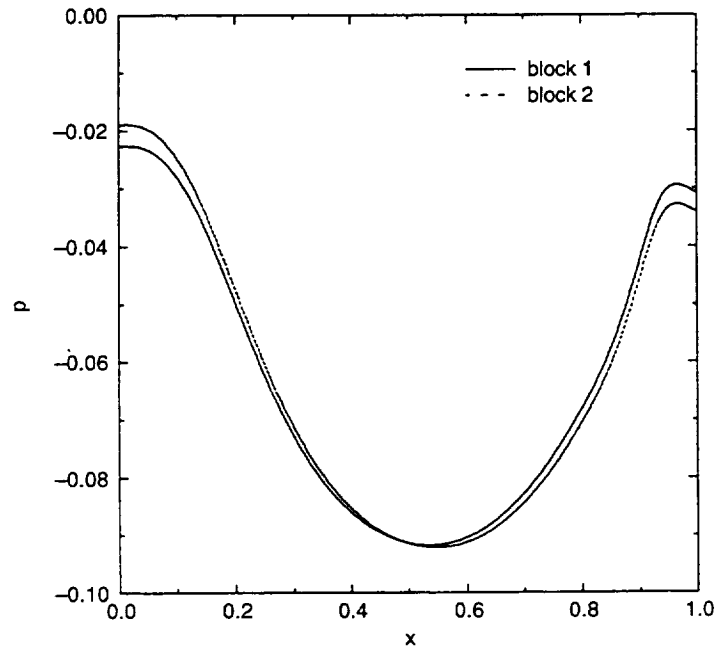


Fig. 9c The pressure distribution at the interface between block 1 and 2 on the fine grid system.

Fig. 9 Solution profiles based on quadratic interpolation for U and V without mass conservation correction.

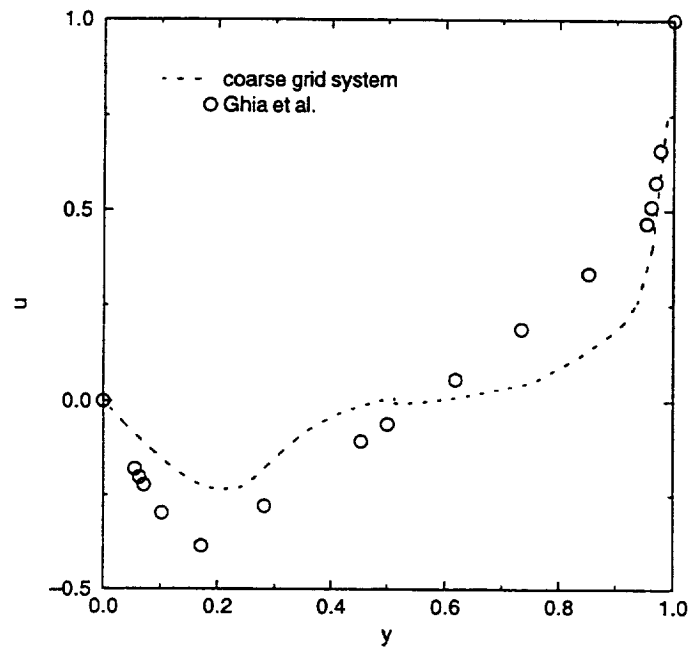


Fig. 10a The u velocity along the vertical center line

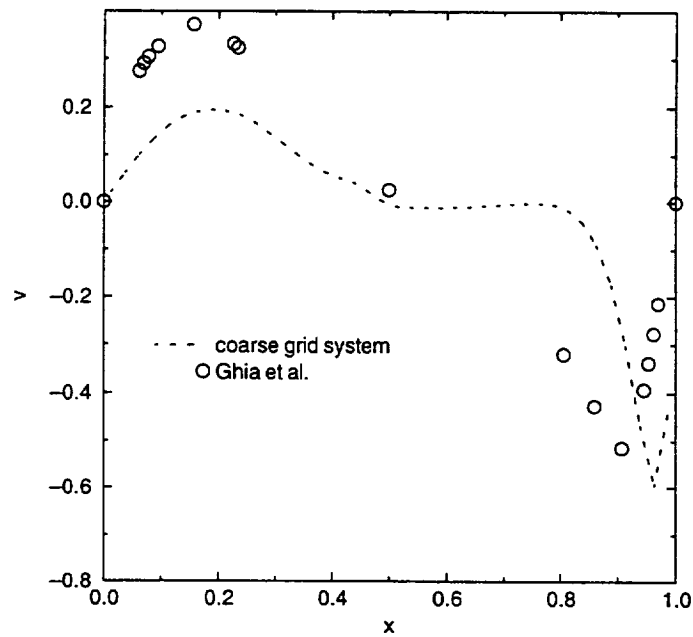


Fig. 10b The v velocity along the horizontal center line.

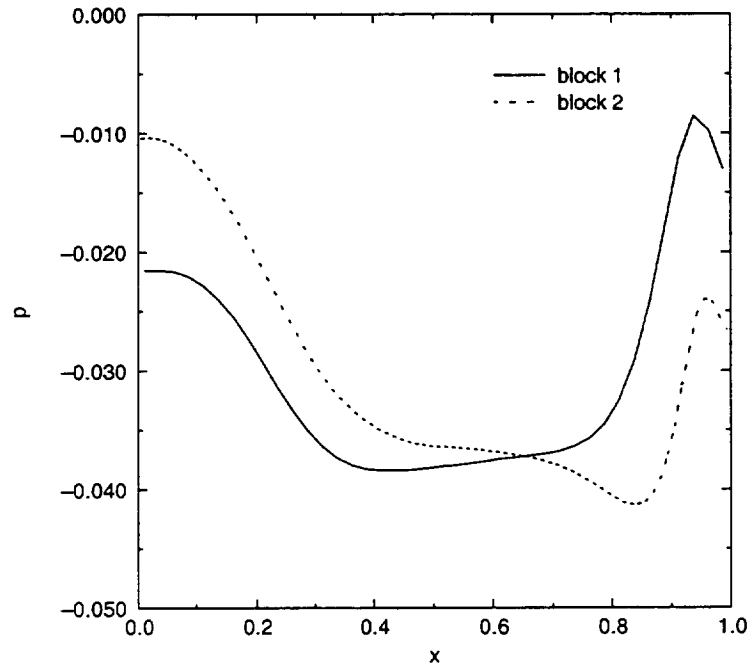


Fig. 10c The pressure distribution at the interface between block 1 and 2 on the coarse grid system.

Fig. 10 Solution profiles based on linear interpolation with global correction of mass conservation for U and V on the coarse grid system.

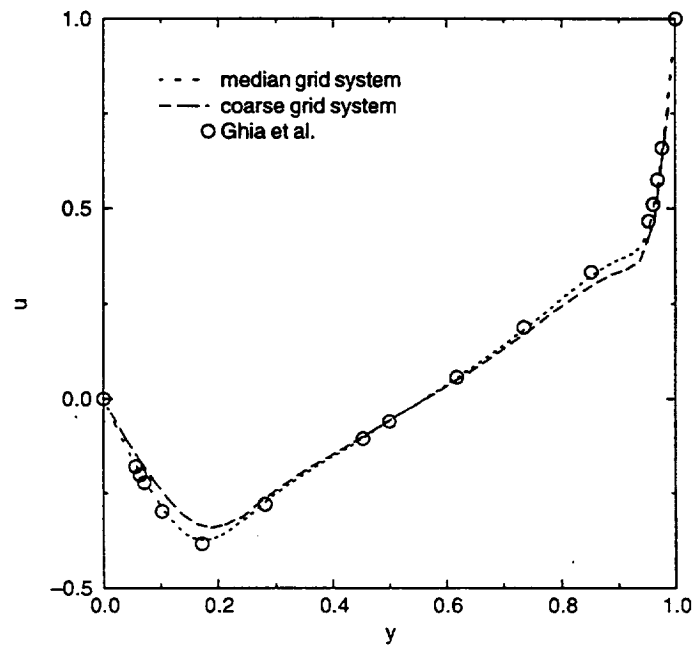


Fig. 11a The u velocity along the vertical center line.

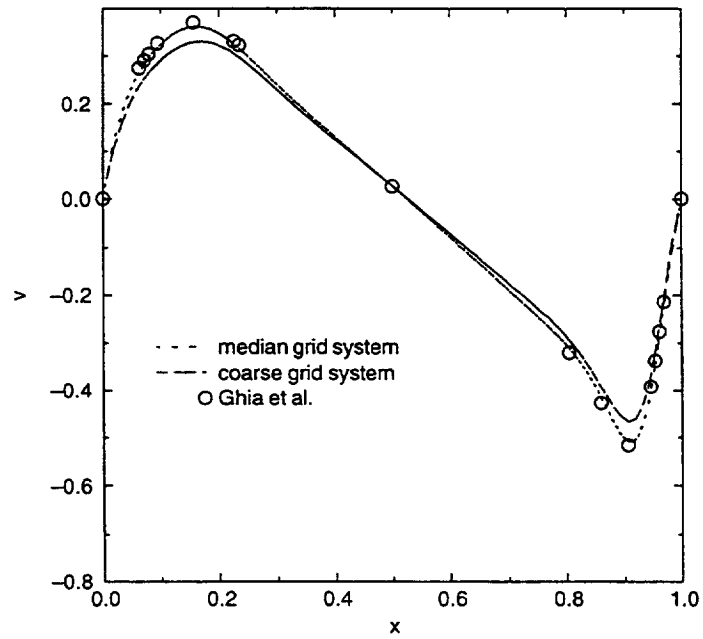


Fig. 11b The v velocity along the horizontal center line.

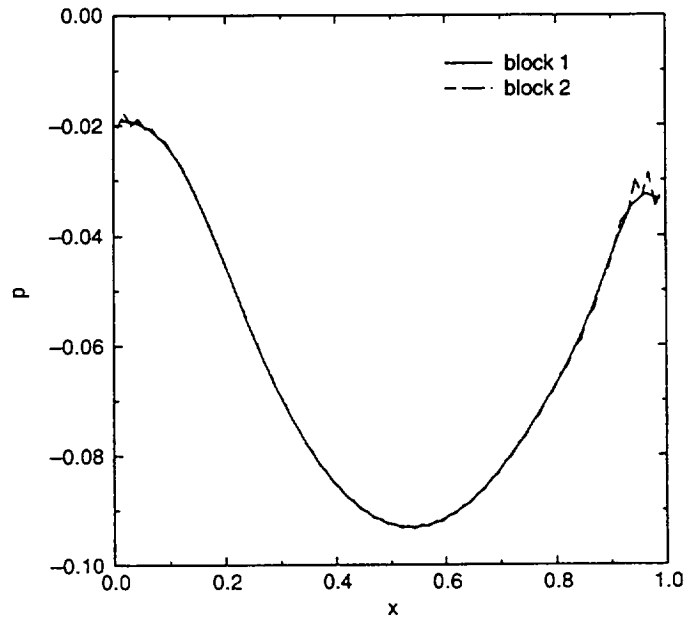


Fig. 11c The pressure distribution at the interface between block 1 and 2 on the coarse grid system.

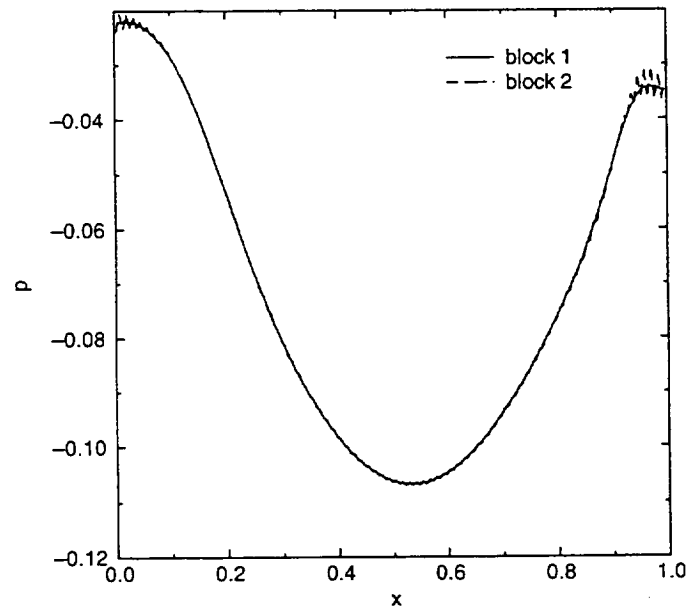


Fig. 11d The pressure distribution at the interface between block 1 and 2 on the median grid system.

Fig. 11 Solution profiles based on piecewise constant interpolation for U and V.

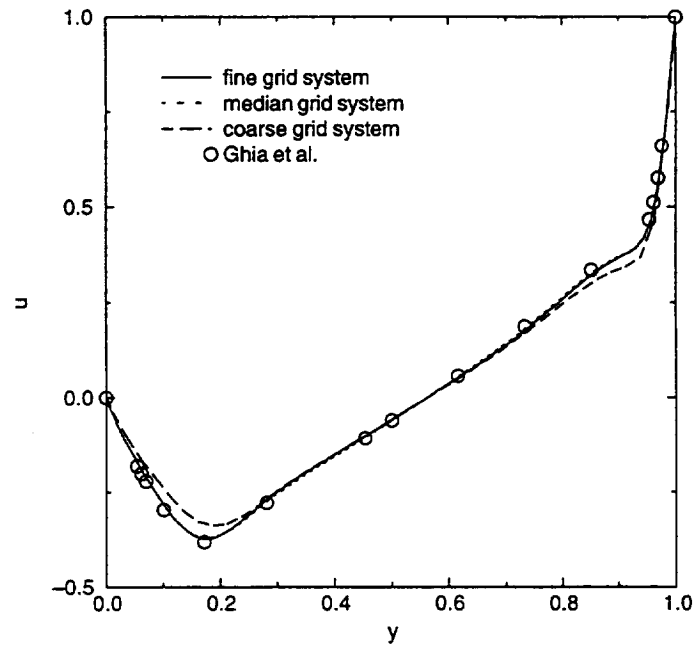


Fig. 12a The u velocity along the vertical center line.

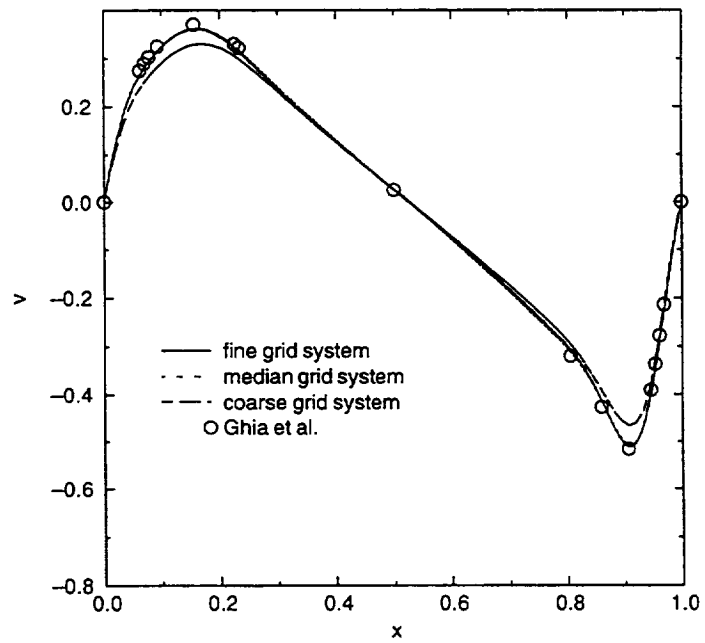


Fig. 12b The v velocity along the horizontal center line.

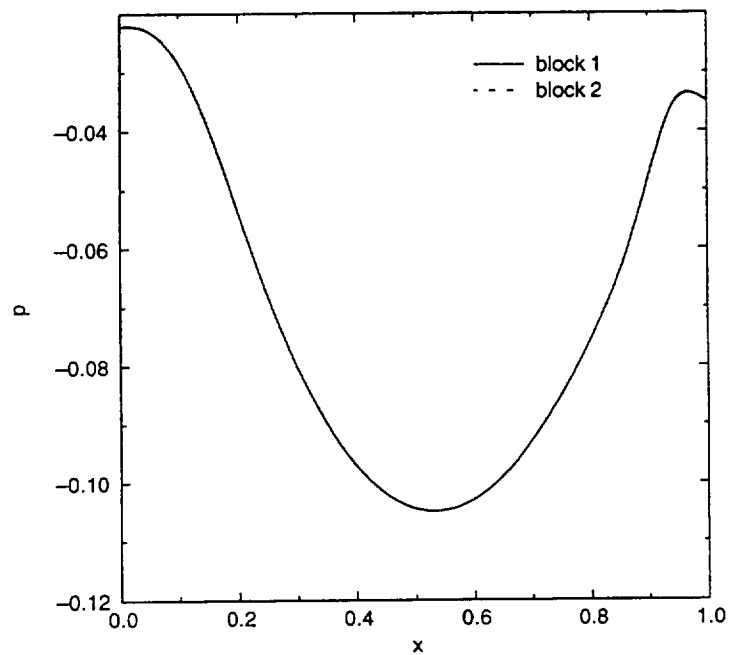


Fig. 12c The pressure distribution at the interface between block 1 and 2 on the fine grid system.

Fig. 12 Solution profiles based on linear interpolation with local correction of mass conservation for U and V.

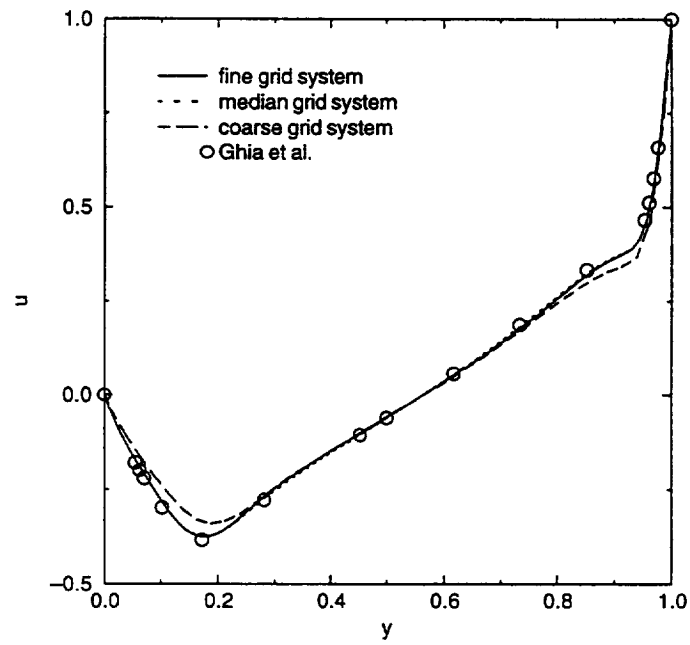


Fig. 13a The u velocity along the vertical center line.

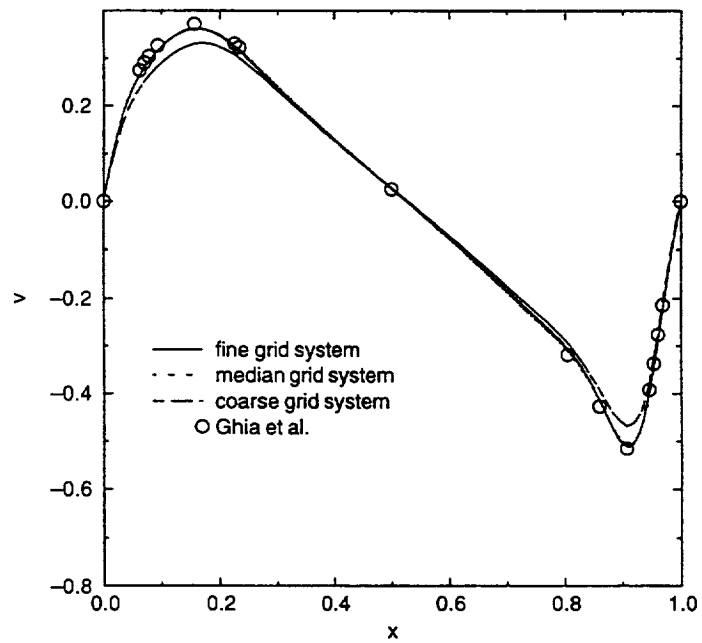


Fig. 13b The v velocity along the horizontal center line.

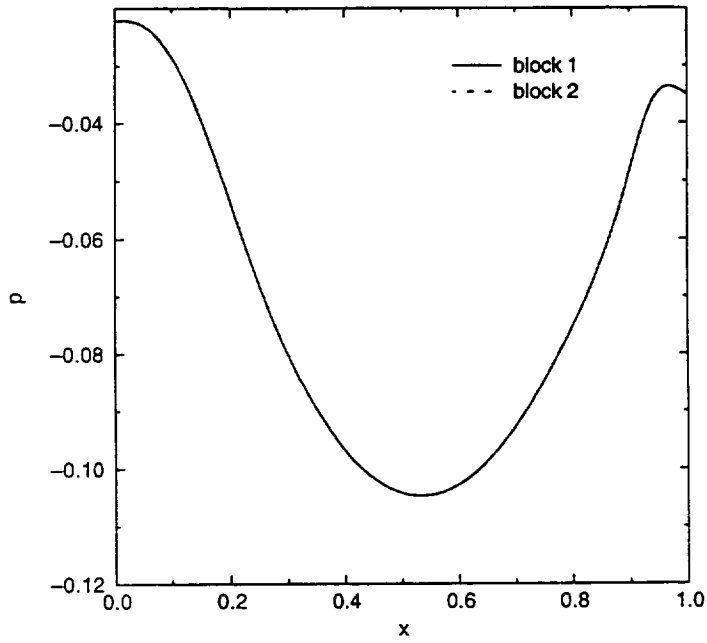


Fig. 13c The pressure distribution at the interface between block 1 and 2 on the fine grid system.

Fig. 13 Solution profiles based on quadratic interpolation with local correction of mas conservation for U and V

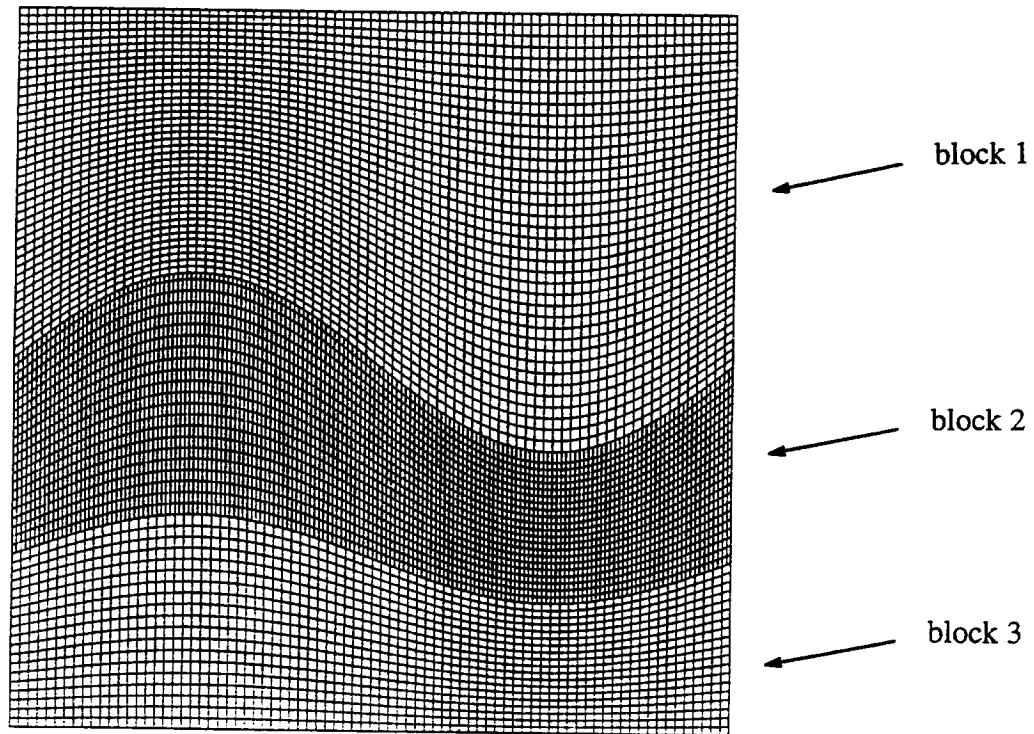


Fig. 14 Three-block curvilinear grid with 81x41, 161x23 and 81x21 nodes for block 1, 2 and 3, respectively.

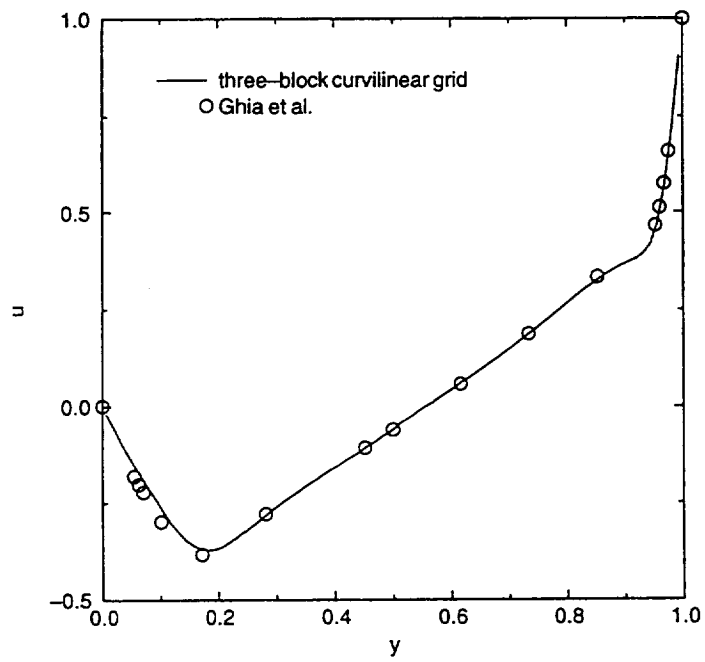


Fig. 15 The u velocity along the vertical center line. Linear interpolation with local correction of mass conservation for U and V .

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